A physics-based integer-linear battery modeling paradigm

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A B S T R A C T

Optimal steady-state dispatch of a stand-alone hybrid power system determines a fuel-minimizing distribution strategy while meeting a forecasted demand over six months to a year. Corresponding optimization models that integrate hybrid technologies such as batteries, diesel generators, and photovoltaics with system interoperability requirements are often large, nonconvex, nonlinear, mixed-integer programming problems that are difficult to solve even using the most state-of-the-art algorithms. The rate-capacity effect of a battery causes capacity to vary nonlinearly with discharge current; omitting this effect simplifies the model, but leads to over-estimation of discharge capabilities. We present a physics-based set of integer-linear constraints to model batteries in a hybrid system for a steady-state dispatch optimization problem that minimizes fuel use. Starting with a nonlinear set of constraints, we empirically derive linearizations and then compare them to a commonly used set of constraints that assumes a fixed voltage and capacity. Numerical results demonstrate that assuming a fixed voltage and capacity may lead to over-estimating discharge quantities by up to 16% compared to our overestimations of less than 1%.

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1. Introduction

Optimal steady-state dispatch of a stand-alone hybrid power system determines a fuel-minimizing power distribution strategy while meeting a forecasted demand over six months to a year. Corresponding optimization models that integrate hybrid technologies such as batteries, diesel generators, and photovoltaics (PV) with system interoperability requirements are often large, nonconvex, nonlinear, mixed-integer programming (MINLP) problems that are difficult to solve [5,29]. Mixed-integer programs (MIPs) that employ linearization and approximation techniques to bound and solve a simplified nonlinear problem frequently serve as tractable modeling alternatives to MINLPs. Although a MIP does not guarantee the feasibility and/or optimality of the solution to the corresponding nonlinear problem, it can still be an effective modeling option.

Batteries are integral to hybrid systems because of their ability to both store energy and provide on-demand, dispatchable power [10,21]. There are a variety of ways to effectively dispatch batteries in hybrid systems such as load-leveling, peak-shaving, and/or load-following [7], all of which store energy to minimize the use of additional technologies with start-up and ramping costs. Subject to a power loss of up to 10%, all batteries generate a direct-current (DC) power that requires conversion to alternating-current (AC) power to meet demand. Batteries store energy in electrochemical form in ways that differ by type; common battery chemistries associated with hybrid systems are nickel cadmium, nickel-metal hydride, lead acid, and lithium-ion. Each has its own documented advantages [35,39], e.g., cost, safety, performance, and size. We
use data from lithium-ion and lead-acid battery manufacturers to represent baseline products within the industry.

Steady-state dispatch modeling for hybrid-power systems often assumes that the instantaneous fluctuations in frequency and voltage over time do not affect power dispatch. When a battery discharges, the electro-active species oxidizes at the anode and reduces at the cathode. The chemistry of the electrodes determines the voltage of an individual cell. Because concentrations of the electro-active species play a large role in determining the voltage and capability of a battery, detailed battery models often track concentrations in various parts of the battery. These diffusion-based models are accurate, because they take into account the transient voltage behavior of the battery, such as the voltage profile when switching between charge and discharge. The time for diffusion, i.e., the dissipation of the concentration gradient, is estimated by the quotient of the square of the battery electrolyte thickness and a coefficient that accounts for the diffusivity of the active species. The diffusivity of lithium ions in the electrolyte is on the order of $10^{-10}$ m$^2$/s, while the scale for different battery components is around 100 µm. This gives diffusion-based models a timescale, length$^2$/diffusivity, on the order of 100 s, which, when compared at the hourly time fidelity of our steady-state application, represents too short of a time interval for it to impact the accuracy of the model.

Although steady-state assumptions facilitate simplifying some complex relationships through linearization [8,13], modeling battery performance often requires both nonlinear and integer considerations. We recognize that lifetime is also an important characteristic in battery modeling; however, lifetime data is limited and difficult to properly test. The focus of this paper is on the set of constraints within a hybrid system optimization model that dictate battery performance, which are nonlinear. For more details, we refer the reader to [33]. Energy, which describes the total amount of electricity a source can provide, and power, which represents the total energy consumed per unit time, are functions of voltage. Although voltage varies with time, our steady-state assumptions render this variance negligible. However, voltage also varies depending on state of charge (SoC), i.e., the fraction of battery capacity available for discharge, and current; failing to consider this variability causes an over-estimation of battery resource availability. By employing a physics-based definition, in which average power is the product of current and voltage, and Ohm's law, which relates current and resistance to voltage, we can model the rate and associated amount of electricity available from batteries in terms of a single independent variable, i.e., current.

A battery’s capacity varies based on the magnitude of the discharge current. The higher the current draw, the less total capacity available, which we refer to as the rate-capacity effect. Peukert’s equation [12], which accounts for the rate-capacity effect in lead-acid batteries by exponentially relating battery capacity to discharge current, could also be applied to other battery chemistries. The Kinetic Energy Battery Model (KiEBM), which portrays the change in capacity as a nonlinear function of charge and discharge rates, models a battery’s capacity as two tanks, one of which is immediately available for discharge and the other of which is chemically bound [22]. Alternatively, the CIMEA (Centre for Energy, Environment and Technology in Madrid, Spain) model presents a nonlinear set of equations that accounts for dynamic and complex battery operations [9]. The common theme among these battery representations is that neglecting rate-capacity leads to overestimating the performance of the battery, which may yield an infeasible dispatch solution, i.e., battery discharge power that is not actually available. Researchers tend to use constraint sets in a hybrid model that neglect rate-capacity (see Section 2 for cases in which tractability is a concern). These large-scale MIPs minimize costs in design and/or dispatch problems that determine optimal technology procurement and/or operation to satisfy demand [6,16–20,25,27,30,31,34,37,38]; they may also approximate unit commitment that schedules technologies to meet demand [15,23,36].

Recognizing the pitfalls of over-simplifying battery performance and the potential impact rate-capacity error has on the solution, we first introduce simple energy constraint set ($\xi$), which not only neglects rate-capacity effects, but assumes a constant voltage. As an alternative to ($\xi$), we present a nonlinear set of constraints ($\lambda$). Next, we linearize the nonlinear, nonconvex relationships in ($\lambda$) to empirically derive a physics-based set of constraints ($P_0^\theta$). We then determine the theoretical error associated with both ($\xi$) and ($P_0^\theta$). Lastly, we present ($P_\theta^\theta$) (see Appendix), which includes a fuel-minimizing objective function and a set of mixed-integer constraints for system interoperability and PV and generator technologies. The combination of ($P_0^\theta$) with ($\xi$) and ($P_\theta^\theta$) forms two optimization models, which we term ($P_\theta^\theta$) and ($\xi$), respectively, for the hybrid power steady-state dispatch problem. We solve 12 scenarios and compare battery dispatch solutions of each model by quantifying the error in each.

The remainder of this paper is organized as follows: Section 2 presents ($\xi$); Section 3 reveals our physics-based mixed-integer, nonlinear battery constraints ($\lambda$) presented in MINLP format. Section 4 details how we derive ($P_\theta^\theta$) from ($\lambda$). Section 5 presents scenarios and results, while Section 6 concludes.

2. A commonly used set of battery constraints ($\xi$)

In this section, we present a commonly employed set of mixed-integer battery constraints ($\xi$) in which voltage is constant and SoC serves as an unrestricted proxy for the fraction of capacity available for discharge. We use lower-case letters for parameters and upper-case letters for variables. We also use lower-case letters for indices and upper-case script letters for sets. Superscripts and accents distinguish between parameters and variables that utilize the same base letter, while subscripts identify elements of a set. The units of each parameter and variable are provided in brackets after its definition, where applicable.

| Sets | $t \in T$ a single time period within the set of time periods |
| Parameters | $\tau$ length of one time period [h]  
$\hat{e}$ manufacturer energy maximum rated-capacity of the battery [W h]  
$\eta^+,\eta^-$ conversion efficiency of power flow into and out of the battery, respectively |
| Variables | $B^{\text{SoC}}_t$ SoC of the battery in time period $t$  
$P^+_t, P^-_t$ aggregate power into and out of the battery in time period $t$, respectively [W]  
$B^+_t, B^-_t$ 1 if the battery is charging or discharging, respectively, in time period $t$, 0 otherwise |

<table>
<thead>
<tr>
<th>Constraints ($\xi$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{e}B^{\text{SoC}}<em>t = \dot{e}B^{\text{SoC}}</em>{t-1} + \tau(\eta^+P^+_t - P^-_t)$ $\forall t \in T : t &gt; 1$ (1a)</td>
</tr>
<tr>
<td>$\tau P^+_t \leq \dot{e}B^{\text{SoC}}_t \forall t \in T : t &gt; 1$ (1b)</td>
</tr>
<tr>
<td>$\tau P^-_t \leq \dot{e}(1 - B^{\text{SoC}}_t) \forall t \in T : t &gt; 1$ (1c)</td>
</tr>
<tr>
<td>$\tau P^+_t \leq \dot{e}B^+_t \forall t \in T$ (1d)</td>
</tr>
<tr>
<td>$\tau P^-_t \leq \dot{e}B^-_t \forall t \in T$ (1e)</td>
</tr>
<tr>
<td>$B^{\text{SoC}}_t \leq 1 \forall t \in T$ (1f)</td>
</tr>
</tbody>
</table>
3. A nonlinear set of battery constraints (∈Ω)

In this section, we present a set of nonlinear, mixed-integer constraints (∈Ω) for modeling battery dispatch in a hybrid system to meet demand. For an algorithmic interpretation of these constraints, we refer the reader to [32]. Throughout this section, we apply notational conventions similar to those in (€).

Sets
\[ t \in \mathcal{T} \] a single time period within the set of time periods
\[ h \in \mathcal{H} \] a single iteration within the set of iterations

Parameters
\( \tau \) length of one time period [h]
\( a^\nu, b^\nu \) voltage slope and intercept coefficients, respectively [V]
\( c_{\text{ref}} \) manufacturer-specified capacity of the battery [Ah]
\( d_t \) power demand in time period \( t \) [W]
\( I \) maximum current allowed [A]
\( p_{\text{ref}} \) reference current of the battery [A]
\( \eta^+, \eta^- \) conversion efficiency of power flow into and out of the battery, respectively
\( p \) Peukert’s constant
\( r_{\text{int}} \) internal resistance of the battery [Ohm]
\( s \) maximum allowed SoC

Variables
\( B_{\text{soc}}^t \) SoC of the battery in time period \( t \)
\( \hat{B}_{\text{soc}}^t \) adjusted SoC of the battery in time period \( t \)
\( \hat{C}_t \) adjusted capacity of the battery in time period \( t \) [Ah]
\( R_{\text{int}}^t \) internal resistance of the battery in time period \( t \) [Ohm]
\( I_t^+, I_t^- \) charge and discharge current of the battery in time period \( t \), respectively [A]
\( P_t^+, P_t^- \) aggregate power into and out of the battery in time period \( t \), respectively [W]
\( V_t^+, V_t^- \) charge and discharge terminal voltage of the battery in time period \( t \), respectively [V]
\( B_t^+, B_t^- \) 1 if the battery is charging or discharging, respectively, in time period \( t \), 0 otherwise

3.1. Nonlinear battery modeling

We model average battery power output as a function of the physics-based nonlinear, nonconvex relationship between current and voltage, which, in general form for discharge, is:

\[ P_t^- = I_t^- V_t^- \quad \forall t \in \mathcal{T} \quad (2a) \]

Here \( V_t^- \) represents the terminal voltage of the battery, which is not constant throughout discharge. Open circuit voltage is a function of SoC and is often modeled as a linear relationship [26]. Terminal voltage deviates from open circuit voltage, which is a linear function of SoC, due to the internal resistance (from Ohm’s law) of the battery (positive for charge, negative for discharge) [3,9,22,28].

\[ V_t^- = a^b_{\text{soc}} B_{\text{soc}}^{t-1} + b^b - B_{\text{soc}}^t I_t^- \quad \forall t \in \mathcal{T} : t > 1 \quad (2b) \]

A battery may be able to support a high-discharge current, but the corresponding capacity achieved is less than the capacity achieved at a lower current. This effect can be empirically described by Peukert’s nonlinear equation, which gives an adjusted capacity \( \hat{C}_t \) based on an applied current \( I_t^- \) [12].

\[ \hat{C}_t = c_{\text{ref}} \left( \frac{I_t^-}{I_t^+} \right)^{p-1} \quad \forall t \in \mathcal{T} \quad (2c) \]

The rate-capacity and adjusted capacity are temporary effects. At the end of a high-discharge current, a battery may appear empty but can actually be discharged further by reducing the current. This recovery effect occurs primarily in lithium-ion batteries and is a result of concentration gradients building up in the cell. Fig. 1 shows that the final capacity achieved stays constant despite high-rate discharges giving lower capacities.

To monitor the capacity available, we introduce adjusted SoC, \( \hat{B}_{\text{soc}}^t \), which relates adjusted capacity to the battery’s SoC and discharge current (capacity consumed).

\[ \hat{B}_{\text{soc}}^t = \frac{\hat{C}_t - c_{\text{ref}} \left( 1 - B_{\text{soc}}^{t-1} \right) - \tau I_t^-}{\hat{C}_t} \quad \forall t \in \mathcal{T} : t > 1 \quad (2d) \]

If the adjusted SoC is greater than or equal to 0, then the discharge current is feasible, which, when we use the right-hand side of (2d) to represent \( \hat{B}_{\text{soc}}^t \), yields the following condition for feasibility:

\[ \hat{C}_t - c_{\text{ref}} \left( 1 - B_{\text{soc}}^{t-1} \right) - \tau I_t^- \geq 0 \quad \forall t \in \mathcal{T} : t > 1 \quad (2e) \]

We use an ampere-hour counting method to increase, at fractional rate \( \eta^- \), or decrease the previous time period’s SoC by the quotient of the current passed during each time period and the capacity [3,8,28]. Discharge efficiency \( \eta^- \) applies to system interoperability constraints, so we exclude it here.

\[ B_{\text{soc}}^t = B_{\text{soc}}^{t-1} + \tau \left( \frac{\eta^- I_t^- - I_t^+}{c_{\text{ref}}} \right) \quad \forall t \in \mathcal{T} : t > 1 \quad (2f) \]

3.2. A mixed-integer, nonlinear set of battery-only constraints (∈Ω)

We now present (∈Ω), a mixed-integer nonlinear set of battery-only constraints for a hybrid optimization model, which provides a more comprehensive dispatch methodology to solving this problem as it considers the entire time horizon simultaneously.

Constraints (∈Ω)

\[ P_t^+ = (a^b_{\text{soc}} B_{\text{soc}}^{t-1} + b^b - B_{\text{soc}}^t I_t^+) \quad \forall t \in \mathcal{T} : t > 1 \quad (3a) \]

\[ P_t^- = (a^b_{\text{soc}} B_{\text{soc}}^{t-1} + b^b - B_{\text{soc}}^t I_t^-) \quad \forall t \in \mathcal{T} : t > 1 \quad (3b) \]

\[ \hat{C}_t = \left( \frac{c_{\text{ref}} \left( \frac{I_t^-}{I_t^+} \right)^{p-1}}{\hat{C}_t} \right)^{\hat{B}_{\text{soc}}^t} \quad \forall t \in \mathcal{T} \quad (3c) \]

\[ \left( \frac{\hat{C}_t - c_{\text{ref}} \left( 1 - B_{\text{soc}}^{t-1} \right) - \tau I_t^-}{\hat{C}_t} \right) B_t^- \geq 0 \quad \forall t \in \mathcal{T} : t > 1 \quad (3d) \]
$I_1^t < IB_1^t \quad \forall t \in T$  \hspace{2cm} (3e)
$I_2^t < IB_2^t \quad \forall t \in T$  \hspace{2cm} (3f)
$B_{soc}^t \leq 1 \quad \forall t \in T$  \hspace{2cm} (3g)
$B_{soc}^t = B_{soc}^{t-1} + \tau \left( \frac{I_1^t - L}{C_{soc}} \right) \quad \forall t \in T : t > 1$  \hspace{2cm} (3h)
$B_1^t + B_2^t \leq 1 \quad \forall t \in T$  \hspace{2cm} (3i)
$B_{soc}^t, C_t, I_1^t, P_1^t, P_2^t, R_{int}^{soc} \geq 0 \quad \forall t \in T$  \hspace{2cm} (3j)
$B_1^t, B_2^t \text{ binary} \quad \forall t \in T$  \hspace{2cm} (3k)

Eqs. (3a) and (3b) determine the power entering and leaving the battery as the product of current and voltage, respectively (see (2a) and (2b)). We do not explicitly model voltage in these constraints as it is a function of current and SoC. Eq. (3c) adjusts the battery’s capacity to account for the discharge current (see (2c)). Constraint (3d) also only applies to discharge (see (2d) and (2e)); if the left-hand side is greater than or equal to zero, then the discharge current is feasible. Constraints (3e) and (3f) bound charge and discharge current, respectively, by relating them to their respective binary variables, while (3g) restricts SoC to a maximum value. In Eq. (3h), we update the SoC (see (2f)). We include $\eta$ when we integrate ($\mathcal{N}$) with ($\mathcal{P}$'), because the latter model contains system interoperability, specifically, power balance constraints. Constraint (3i) prevents the battery from simultaneously charging and discharging during a given time period; it also allows the battery to be idle. Lastly, constraints (3j) and (3k) provide nonnegativity and binary restrictions, respectively.

We do not attempt to solve a hybrid optimization model with ($\mathcal{N}$), because its nonlinearity and nonconvexity lead to tractability issues even if the domain violations in constraints (3c) and (3d) are removed. Although constraints (3a)–(3d) are nonlinear, these nonlinearities are linearizable by employing known linearization techniques and empirical approximations, as we show in the next section.

4. A physics-based linear set of battery constraints ($\mathcal{P}^p$)

In this section, we demonstrate how to approximate the following operational characteristics present in ($\mathcal{N}$): (i) voltage that varies with current and the previous time period’s SoC, and (ii) capacity that varies exponentially with discharge current. Throughout this section, we apply similar notational conventions as we do to ($\mathcal{E}$).

4.1. Voltage

Similar to constraint (2a), we model average battery power output as a function of the physics-based nonlinear, nonconvex relationship between current and voltage (see (2b)). Voltage drops that occur during charge or discharge result from the internal resistance and concentration gradients that occur during use. Internal resistance is determined by the properties of the electrolyte and the distance between the two electrodes, and is often portrayed as a simple series resistance following Ohm’s law. As the battery charges or discharges, ions build up at the electrode and electrolyte interfaces, which also causes the voltage to change. Both concentration gradient and internal resistance offsets depend on the applied current and usually remain constant throughout discharge ($R_{int}^{soc} = R_{int}$). When the battery discharges, the terminal voltage drops below the open circuit value. When the battery is recharged, the terminal voltage is greater than the open circuit value. The hourly time fidelity of our application allows us to ignore transient features, such as the build-up of concentration gradients, and to model discharge voltage as a function of open circuit voltage and resistance [9].

We approximate current in (2b) with $r^{soc}$, which represents a typical current applied by the battery. The voltage offset due to Ohm’s Law, then, simplifies to a single constant resistance term $r^{soc} r^{soc}$, which we add to the open circuit voltage for charge and subtract for discharge. The error associated with this approximation is discussed in detail in Section 5. Fig. 2 displays this relationship.

Substituting voltage relationships into Eq. (2a) and the analogous equation for charge to determine power entering and exiting the battery yields:

$$P_t^- = (a_t B_{soc}^t + b_t + r^{soc} r^{soc}) I_t^- \quad \forall t \in T : t > 1$$  \hspace{2cm} (4a)
$$P_t^+ = (a_t B_{soc}^t + b_t - r^{soc} r^{soc}) I_t^+ \quad \forall t \in T : t > 1$$  \hspace{2cm} (4b)

The bilinear relationships within Eqs. (4a) and (4b) between SoC and current are linearizable [33] using a convex-underestimation technique introduced by [4,24] to bound and solve the problem.
4.2. Capacity

A battery realizes its maximum capacity when discharges occur at low currents, because concentration gradients and the resulting voltage shifts are small. At high currents, the capacity realized may be less than half of that obtained at lower currents. We call the nonlinear relationship between capacity and discharge current the rate-capacity effect. Battery capacity as a function of current can often be approximated using an exponential relationship (see Peukert’s equation (3c)). To facilitate tractability of the problem, we use a linear relationship, based on specific rate-capacity data, to approximate capacity as a function of current in which the reference capacity $c^{ref}$ represents the $y$-intercept and $c$ represents the slope of the line ($c < 0$).

\[ \dot{C}_t = c^{ref} + c \dot{I}_t \quad \forall t \in T \]

(4c)

\[ l \leq \dot{I}_t \leq 1 \quad \forall t \in T \]

(4d)

Fig. 3 presents capacity data for a lead-acid battery. We fit Peukert’s nonlinear empirical equation (3c) and our linear approximation (4c) employing least-squares regression to a range of feasible current values given in constraint (4d).

To determine a rate-capacity bound, i.e., to restrict current to a feasible range, we substitute the linear approximation (4c) for $\dot{C}_t$ in (2e) and then solve for $\dot{I}_t$, which represents an upper-bound, so we replace the equality with an inequality:

\[ \dot{I}_t \leq \left( \frac{c^{ref}}{\tau - c} \right) B^\text{soc} \quad \forall t \in T : t > 1 \]

(4e)

4.3. Formulation

In this section, we present (\text{P}^\text{eq}) as a mixed-integer set of constraints derived in Sections 4.1 and 4.2. We apply similar notational conventions to (\text{P}^\text{eq}) as we applied to both (\text{E}) and (\text{N}).

**Additional parameters**

- $c^{ref}$: battery discharge capacity slope coefficient [h]
- $\bar{I}$: predicted average current of the battery [A]
- $\underline{I}$: minimum allowed current [A]
- $\underline{s}$: minimum allowed SoC

**Constraints (\text{P}^\text{eq})**

\[ P^1_t = (a^1 B^\text{soc} + b^1 + \tau^{\text{pred}}) \bar{I} \quad \forall t \in T : t > 1 \]

(5a)

\[ P^2_t = (a^2 B^\text{soc} + b^2 - \tau^{\text{pred}}) \bar{I} \quad \forall t \in T : t > 1 \]

(5b)

\[ \dot{I}_t \leq \left( \frac{c^{ref}}{\tau - c} \right) B^\text{soc} \quad \forall t \in T : t > 1 \]

(5c)

\[ \underline{s} \leq B^\text{soc} \leq \bar{s} \quad \forall t \in T \]

(5d)

\[ \underline{B}^\text{soc} \leq \dot{I}_t \leq \dot{B}^\text{soc} \quad \forall t \in T \]

(5e)

\[ \underline{B}^\text{soc} \leq \dot{I}_t \leq \dot{B}^\text{soc} \quad \forall t \in T \]

(5f)

\[ B^\text{soc} = B^\text{soc} + \tau \left( \frac{\eta \dot{I}_t - \dot{B}^\text{soc}}{c^{ref}} \right) \quad \forall t \in T : t > 1 \]

(5g)

\[ B^\text{soc} + B^\text{soc} \leq P^1_t \quad \forall t \in T \]

(5h)

\[ B^\text{soc} \geq 0 \quad \forall t \in T \]

(5i)
Eq. (5d) restricts the SoC of the battery to a range that supports our linear assumptions (see Fig. 2), while constraints (5e) and (5f) do the same for charge and discharge current, respectively (see constraint (4d)). Constraints (5g)–(5j) follow identically from (1a)–(1d).

5. Theoretical error analysis

To effectively compare the two sets of battery constraints, we quantify the approximation error associated with linearizing (7c) and compare it to the rate-capacity and constant voltage assumption error in solutions to (7). We define the following variables to facilitate this analysis:

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_t^c )</td>
<td>magnitude of error in time period ( t ) per feasible battery dispatch according to (7c) [W]</td>
</tr>
<tr>
<td>( \delta_t^{\text{rpt}} )</td>
<td>magnitude of error in time period ( t ) per feasible battery dispatch according to (7b) [W]</td>
</tr>
</tbody>
</table>

5.1. Rate-capacity and constant voltage assumption error in (7c)

In (7c), the capacity of the battery, as given by the product of its maximum energy \( \epsilon \) rating and SoC, is 100\% available for discharge (see constraint (1b)). Based on our discussion of capacity and current in Sections 1 and 3, only a fraction of the battery’s capacity is actually available for discharge. Given that (7c) assumes a constant voltage, and given our use of the physics-based definition of power as a function of voltage and current, any bound on power in (7c) also restricts current. In (4e), we derive a rate-capacity bound \( \epsilon^{\text{c}} \mathbf{B}_{\text{soc}} \) that relates current to SoC and capacity. The difference between this bound and constraint (1b) is the rate-capacity multiplier \( 1 / (t - c^{-}) \), which we label as \( \epsilon^{c} \) [h^{-1}]. If we divide both sides of constraint (1b) by \( t \) and then replace the multiplier \( 1 / c \), which assumes no rate-capacity effects, on the right-hand side with \( \epsilon^{c} \), rate-capacity bounds on discharge power follow as:

\[
P_t \leq \epsilon^{c} \mathbf{B}_{\text{soc}}^{\text{rpt}} \quad \forall t \in T : t > 1
\]  

(6a)

Table 1 provides values of the parameter \( \epsilon^{c} \) and available capacity percentages for one lead-acid battery and five lithium-ion batteries. Even within the same chemistry, batteries can behave differently based on cell construction and active materials. Even within the same chemistry, batteries can behave differently based on cell construction and active materials.

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Chemistry</th>
<th>( \epsilon^{c} )</th>
<th>Available capacity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>K2 18650</td>
<td>Lithium-Ion</td>
<td>−0.009</td>
<td>99.1</td>
</tr>
<tr>
<td>A123 12-Volt</td>
<td>Lithium-Ion</td>
<td>−0.04</td>
<td>96.1</td>
</tr>
<tr>
<td>Panasonic NCR 18650</td>
<td>Lithium-Ion</td>
<td>−0.08</td>
<td>92.5</td>
</tr>
<tr>
<td>Panasonic GCR 18650</td>
<td>Lithium-Ion</td>
<td>−0.27</td>
<td>78.7</td>
</tr>
<tr>
<td>Panasonic 18650</td>
<td>Lithium-Ion</td>
<td>−0.47</td>
<td>68.0</td>
</tr>
<tr>
<td>Hardy 12-Volt</td>
<td>Lead-Acid</td>
<td>−0.62</td>
<td>61.7</td>
</tr>
</tbody>
</table>

5.2. Approximation error in (7b)

In Section 3.4 of [33], the authors indicate that the discharge approximation error in (7b) stems from the upper bounds imposed by the linearization of current and SoC (\( \mathbf{B}_{\text{soc}}^{\text{rpt}} I_{\text{t}} \)) constraints (5b); the corresponding decrease in SOC for (7c) is given in (1a).

The difference between SOC for (7c) and (1a) multiplied by energy gives the error in watts:

\[
\frac{I_{\text{t}} - P_{\text{t}}}{\epsilon} \quad \forall t \in T
\]  

(6d)

The rate-capacity and constant voltage assumption error in (7c) are additive, but because the constant voltage assumption error may over- or under-estimate power, error in (7c) is not strictly an over-estimation.

\[
\delta_t^c = \max \{ 0, P_t - \epsilon^{c} B_{\text{soc}}^{\text{rpt}} \} + \frac{I_{\text{t}} - P_{\text{t}}}{\epsilon} \quad \forall t \in T
\]  

(6e)

The error associated with charge, rather than discharge, follows similarly.

In SoC in (7c) for a given discharge quantity to that of (1a). We determine the SoC decrease as a function of power discharged in (7c) by multiplying power by \( t / \epsilon \) (see (1a)), yielding \( \tau^{\text{rpt}} P_{\text{t}} / \epsilon \).

To determine the SoC decrease for \( P_{\text{t}} \) in (7c), we first solve Eq. (3b) with \( R_{\text{rpt}} = \epsilon^{\text{rpt}} \) for discharge current, subtracting, rather than adding, the radial in the numerator to remain within allowable bounds:

\[
I_{\text{t}} = \frac{(a^{\text{B}} B_{\text{soc}}^{\text{rpt}} + b^{\text{rpt}}) - \sqrt{(a^{\text{B}} B_{\text{soc}}^{\text{rpt}} + b^{\text{rpt}})^{2} - 4 R_{\text{rpt}}^{2} I_{\text{t}}^{2}}}{2 R_{\text{rpt}}} \quad \forall t \in T
\]  

(6c)

Then, using \( I_{\text{t}} \) from (6c), we determine that the decrease in SoC (1a), which is the quotient of current and capacity, is \( I_{\text{t}} / \epsilon^{\text{rpt}} \) (see (3b)); the corresponding decrease in SOC for (7c) is given in (1a).

The difference between SOC for (7c) and (1a) multiplied by energy gives the error in watts:

\[
\frac{I_{\text{t}} - P_{\text{t}}}{\epsilon} \quad \forall t \in T
\]  

(6d)

The rate-capacity and constant voltage assumption error in (7c) are additive, but because the constant voltage assumption error may over- or under-estimate power, error in (7c) is not strictly an over-estimation.

\[
\delta_t^c = \max \{ 0, P_t - \epsilon^{c} B_{\text{soc}}^{\text{rpt}} \} + \frac{I_{\text{t}} - P_{\text{t}}}{\epsilon} \quad \forall t \in T
\]  

(6e)

The error associated with charge, rather than discharge, follows similarly.

In Section 3.4 of [33], the authors indicate that the discharge approximation error in (7b) stems from the upper bounds imposed by the linearization of current and SoC (\( \mathbf{B}_{\text{soc}}^{\text{rpt}} I_{\text{t}} \)) constraints (5b); the upper bounds resulting from this linearization follow:

\[
\mathbf{B}_{\text{soc}}^{\text{rpt}} I_{\text{t}} \leq \mathbf{B}_{\text{soc}}^{\text{rpt}} \mathbf{I}_{\text{t}} - \mathbf{I}_{\text{t}} \quad \forall t \in T : t > 1
\]  

(7a)

\[
\mathbf{B}_{\text{soc}}^{\text{rpt}} I_{\text{t}} \leq \mathbf{B}_{\text{soc}}^{\text{rpt}} \mathbf{I}_{\text{t}} - \mathbf{I}_{\text{t}} \quad \forall t \in T : t > 1
\]  

(7b)

Because we use \( \tau = 0 \) and \( \tau = 0 \) for the lithium-ion batteries in our scenarios, these constraints simplify to the following:

\[
\mathbf{B}_{\text{soc}}^{\text{rpt}} I_{\text{t}} \leq \mathbf{B}_{\text{soc}}^{\text{rpt}} \mathbf{I}_{\text{t}} - \mathbf{I}_{\text{t}} \quad \forall t \in T : t > 1
\]  

(7c)

\[
\mathbf{B}_{\text{soc}}^{\text{rpt}} I_{\text{t}} \leq \mathbf{I}_{\text{t}} \quad \forall t \in T : t > 1
\]  

(7d)

Constraint (7c) in the approximation is dominated by (5c) in (7b), because \( \epsilon^{\text{rpt}} / (t - c^{-}) = \tau \); therefore, we only use constraint (7d) to determine the error. Moving the left-hand side to the right-hand side of (7d) and factoring out current yields the error as the product of current and the difference between SoC and its upper bound \( (\mathbf{B}_{\text{soc}}^{\text{rpt}}) I_{\text{t}} \), which we multiply by \( a^{\text{B}} \) to convert from amperes to watts. Discharge approximation error follows as:

\[
a^{\text{B}} (\mathbf{B}_{\text{soc}}^{\text{rpt}}) I_{\text{t}} \quad \forall t \in T : t > 1
\]  

(7e)
Additionally, we under-estimate current by approximating it in Eq. (5b) with \( i_{av} = \frac{\text{cref}}{s/C_0/C_1} \), which is the maximum possible discharge quantity per time period. The resulting \( i_{av} \) error is the product of the difference between \( i_{av} \) and current, which, in units of power (W), follows from the last term in Eq. (5b):

\[
\delta_{i_t}^{\text{r}} = \left( a_r (s - B_{soc}^{t+1}) - r^\text{int} \left( \frac{\text{cref}}{\tau} - I \right) \right) I_t \quad \text{for} \quad \tau_t > T \tag{7f}
\]

Given \( \frac{\text{cref}}{\tau} \) is greater than or equal to \( I \), we subtract the \( i_{av} \) error from the discharge error. The combined discharge approximation and \( i_{av} \) error in \( (\text{Pb}) \) for discharge \( \forall t \in T : t > 1 \) is:

\[
\delta_{i_t}^{\text{r,pb}} = \left( a_r (s - B_{soc}^{t+1}) - r^\text{int} \left( \frac{\text{cref}}{\tau} - I_t \right) \right) I_t \quad \text{for} \quad \tau_t > T \tag{7g}
\]

A similar analysis can be done using the lower bounds on charge.

### 5.3. Comparative error analysis of \( \delta_{i}^{\text{r}} \) and \( \delta_{i}^{\text{av}} \)

Although \( \delta_{i}^{\text{r}} \) and \( \delta_{i}^{\text{av}} \) are indexed on \( t, t \) is not necessary for the theoretical analysis. Without loss of generality, we simply address discharge error in this subsection. We compare error as a function of the maximum feasible discharge quantities, which for \( \delta_{i}^{\text{r}} \) is \( P_t = B_{soc}^{t+1}/\tau \) and for \( \delta_{i}^{\text{av}} \) is \( I_t = \frac{\text{cref} B_{soc}^{t+1}/(\tau - c)}{s/C_0/C_1} \) (see Eqs. (1d) and (5c), respectively).

Fig. 4 depicts rate-capacity error in \( \delta_{i}^{\text{r}} \) for each battery listed in Table 1 with a 200 kW h capacity as a function of SoC. Regardless of battery capacity, we find that the greatest errors in solutions to \( \delta_{i}^{\text{r}} \) occur when the battery is fully charged \( (B_{soc} = s) \), but, in general, error increases linearly with SoC. Also, results indicate that as the parameter \( c^r \) increases, rate-capacity error declines, but as \( c^r \) decreases, rate-capacity error increases, i.e., the higher the capacity available per Table 1, the less chance of a rate-capacity violation.

Constant voltage assumption error in \( \delta_{i}^{\text{av}} \) is a function of discharge power \( P_t \) and SoC \( B_{soc}^{t+1} \) (see (6d)). Table 2 displays \( \delta_{i}^{\text{av}} \) for a range of feasible discharge power and SoC values for a 200 kW h battery. Feasibility is determined by imposing rate-capacity bounds in (6a) on power and SoC combinations. Negative values indicate an under-estimation of discharge power for the combination of \( P_t \) and \( B_{soc}^{t+1} \), while positive shaded numbers reflect an over-estimation. Minimum and maximum values are denoted by bold font and are underlined. Empty cells depict infeasible combinations of \( P_t \) and \( B_{soc}^{t+1} \).

#### Table 2

| \( P_t \) (W) | \( B_{soc}^{t+1} \) | \( \delta_{i}^{\text{av}} \)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>0.1</td>
<td>153</td>
</tr>
<tr>
<td>20,000</td>
<td>0.2</td>
<td>345</td>
</tr>
<tr>
<td>30,000</td>
<td>0.3</td>
<td>484</td>
</tr>
<tr>
<td>40,000</td>
<td>0.4</td>
<td>504</td>
</tr>
<tr>
<td>50,000</td>
<td>0.5</td>
<td>536</td>
</tr>
<tr>
<td>60,000</td>
<td>0.6</td>
<td>536</td>
</tr>
<tr>
<td>70,000</td>
<td>0.7</td>
<td>536</td>
</tr>
<tr>
<td>80,000</td>
<td>0.8</td>
<td>536</td>
</tr>
<tr>
<td>90,000</td>
<td>0.9</td>
<td>536</td>
</tr>
<tr>
<td>100,000</td>
<td>1.0</td>
<td>536</td>
</tr>
<tr>
<td>110,000</td>
<td>1.1</td>
<td>536</td>
</tr>
<tr>
<td>120,000</td>
<td>1.2</td>
<td>536</td>
</tr>
<tr>
<td>130,000</td>
<td>1.3</td>
<td>536</td>
</tr>
<tr>
<td>140,000</td>
<td>1.4</td>
<td>536</td>
</tr>
<tr>
<td>150,000</td>
<td>1.5</td>
<td>536</td>
</tr>
<tr>
<td>160,000</td>
<td>1.6</td>
<td>536</td>
</tr>
<tr>
<td>170,000</td>
<td>1.7</td>
<td>536</td>
</tr>
<tr>
<td>180,000</td>
<td>1.8</td>
<td>536</td>
</tr>
<tr>
<td>190,000</td>
<td>1.9</td>
<td>536</td>
</tr>
<tr>
<td>200,000</td>
<td>2.0</td>
<td>536</td>
</tr>
</tbody>
</table>
The maximum discharge error, when we subtract the remaining batteries listed in Table 1, the potential for large errors increases. Total discharge error, which is dominated by the approximation error, is less than 2 kW inaccuracy and solvability. In this section, we present \((P^d)\) (see Appendix A), a mixed-integer, linear optimization model that minimizing fuel consumption subject to a set of constraints that includes system interoperability and bounds for PV and generator technologies. Because the focus of our study lies in battery performance, we eliminate system procurement considerations, including the related aspect of battery lifecycles, from a more comprehensive model [33]. The combination of \((P^d)\) with \((P^f)\) forms two optimization models, \((P'^d)\) and \((P'^f)\), respectively, for the hybrid power steady-state dispatch problem.

We solve these models and compare their solutions by quantifying the error present in each. We employ the following variables:

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_t)</td>
<td>power discharged by the battery in time period (t) per a dispatch solution to ((\xi^d)) [W]</td>
</tr>
<tr>
<td>(P_t^f)</td>
<td>power discharged by the battery in time period (t) per a dispatch solution to ((P^f)) [W]</td>
</tr>
<tr>
<td>(\delta_t^d)</td>
<td>magnitude of error as defined by (\delta_t^d) (see (6e)) in time period (t) per battery dispatch solution to ((\xi^d)) [W]</td>
</tr>
<tr>
<td>(\delta_t'^d)</td>
<td>magnitude of error as defined by (\delta_t'^d) (see (7g)) in time period (t) per battery dispatch solution to ((P'^d)) [W]</td>
</tr>
</tbody>
</table>

6.1. Model parameters and scenarios

We focus on hybrid systems comprised of diesel generators, batteries (energy storage), and PV (renewable energy). All technologies represent baseline industry products in terms of performance and capabilities. We consider three sizes of solar arrays: (i) a 50 kW PV array, which represents roughly half of the maximum demand, (ii) a 100 kW PV array, which represents roughly the same magnitude as the maximum demand, and (iii) a 200 kW PV array, which corresponds to twice the maximum demand. We consider two sizes of generators (see Table 3) and two sizes of Panasonic 18650 batteries, which we assume fully charged and new, i.e., off the shelf. We do not consider terminal conditions of the battery, because our focus is on battery performance constraints for a six-month time horizon, which is not long enough to impact battery lifetime. Correspondingly, we don’t consider battery lifecycles in our analysis. We choose the lithium-ion chemistry because it is the most applicable to our situation in that it has long cycle life, the best energy density, i.e., capacity per weight, and has received the most attention. Without loss of generality, we examine the Panasonic 18650 whose characteristics include available from manufacturers or from other empirical studies to our knowledge. Our battery constraint sets necessitate careful tailoring of existing data, which we consider one of the contributions of our work.

In [33], the authors present 14 year-long demand profiles at hourly-fidelity. We employ the first six months of the San Salvador, El Salvador scenario [14] scaled by 1.15, which, given the cyclical nature of the data, represents a typical steady-state forecast. The minimum demand is 19 kW and the maximum demand is 100 kW.
We model PV power by first determining the AC power output of a 1 kW mono-crystalline panel using a PVWatts simulation for the location from which the demand originates [11]. We then take the product of the PVWatts results and the respective sized PV array (50 kW, 100 kW, or 200 kW) as optimization model input to determine the hourly PV power output. Fig. 8 displays the relationship between demand and PVWatts solar radiation output for the three sizes of PV arrays over a one-week ($|T|=168$) horizon.

Table 3
Generator parameter values from [33] employed in each scenario.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>60 kW</th>
<th>100 kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_f$</td>
<td>0.0645</td>
<td>0.0644</td>
</tr>
<tr>
<td>$c_f$</td>
<td>0.59</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Table 4
Battery parameter values employed in each scenario, in which $r^m$, $P^{ref}$, $a^v$, and $b^v$ derive from relationships defined in Section 4.1, and $r^{crf}$, $c^{crf}$ from linearizations in Section 4.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>100 kW h</th>
<th>200 kW h</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^m$</td>
<td>0.00587</td>
<td>0.00293</td>
</tr>
<tr>
<td>$P^{ref}$</td>
<td>448</td>
<td>897</td>
</tr>
<tr>
<td>$a^v$</td>
<td>10.76</td>
<td>10.76</td>
</tr>
<tr>
<td>$b^v$</td>
<td>217.6</td>
<td>217.6</td>
</tr>
<tr>
<td>$r^{crf}$</td>
<td>448</td>
<td>897</td>
</tr>
<tr>
<td>$c^{crf}$</td>
<td>0.4783</td>
<td>0.4783</td>
</tr>
</tbody>
</table>

Table 5 presents our 12 scenarios, in which each scenario represents a different power-rated combination of PV, generator, and lithium-ion battery.
Given the differences in power capabilities between each scenario, we expect to see a variety of ways in which to dispatch technologies to minimize fuel use. Fig. 9 provides three likely dispatch strategies, which are related to the size of the PV array, for a 24-h period. Fig. 9a displays a likely solution if PV is less than demand (scenarios 1, 2, 7, 8), i.e., PV powers only a fraction of the demand. In these scenarios, the limited availability of PV not only mandates the dispatch of additional technologies to meet demand, but it implies the generator is necessary to charge the battery. Fig. 9b indicates that the increase in PV availability (scenarios 3, 4, 9, 10) allows PV to independently power the load and provide some power to assist the generator in charging the batteries. Fig. 9c demonstrates a dispatch solution for scenarios with an even greater PV availability (scenarios 5, 6, 11, 12) than that exhibited in Fig. 9a and b.

### 6.2. Results

We solve \((E^+)\) and \((PBM^+)\) on a Sun Fire x2270 m2 with 24 processors (2.93 GHz each), 48 GB RAM, 1 TB HDD, using GAMS 24.1.3, which employs CPLEX version 12.5.1.0 [1], a commercial state-of-the-art solver that uses the branch-and-bound algorithm coupled with heuristics to improve the best integer solution and cuts to improve bounds. Each MIP solves to an optimality gap of three percent in fewer than two hours. Table 6 reports the size of the dispatch problem with each set of battery-only constraints.

![Fig. 9](image)

**Table 5**

Each scenario has a diesel generator, PV array, and lithium-ion battery; however, scenarios differ by the power rating of each technology type.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Generator (kW)</th>
<th>Solar array (kW)</th>
<th>Battery (kW h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>4</td>
<td>60</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
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<tr>
<td>6</td>
<td>60</td>
<td>200</td>
<td>100</td>
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<td>7</td>
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<td>50</td>
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</tr>
<tr>
<td>8</td>
<td>100</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>11</td>
<td>100</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>

**Table 6**

Mathematical characteristics, which include number of constraints and variables in \((E^+)\) and \((PBM^+)\) for all scenarios.

<table>
<thead>
<tr>
<th>Constraints</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Continuous</td>
</tr>
<tr>
<td>((E^+))</td>
<td>87,600</td>
</tr>
<tr>
<td>((PBM^+))</td>
<td>170,819</td>
</tr>
</tbody>
</table>

![Fig. 10](image)

Fig. 10 presents the maximum and average of \(d_{E^+}\) and \(d_{PBM^+}\) by scenario. In all but one scenario (Scenario 3), both the maximum and average error of \(d_{PBM^+}\) are notably less than that of \(d_{E^+}\). In this solution, there are no rate-capacity violations so \(d_{E^+}\) equals just the constant voltage assumption error (see Table 2), which is...
comparable in magnitude to discharge approximation error in \( \text{PB}^+ \) (see Fig. 5).

If we separate the 12 scenarios into odd and even sets, in which odd sets correspond to the 200 kW h battery and even sets the 100 kW h battery (see Table 5), and examine the results by set, we see a common theme. Rate-capacity errors occur more frequently and at higher magnitudes when demand is greater than the upper bound on battery discharge. Because the capacity of the battery is twice that of the maximum demand (100 kW) for odd scenarios, rate-capacity errors are small. Fig. 10 reports that the largest violation within the odd set is 9 kW. On the contrary, within the even set, because battery capacity is similar to the maximum demand, the likelihood of a rate-capacity violation increases. In five of the six even scenarios, rate-capacity error is nearly 16 kW and the average of this error across all scenarios is twice that of the odd set. Over-estimating performance by 16% in even scenarios allows a 100 kW h battery to operate as a 116 kW h battery without penalty. However, the procurement cost differential at $500 per kW h between a 100 kW h and 116 kW h battery is $8000. Additionally, a 100 kW generator consumes 2 gallons of fuel in producing 16 kW in an hour, which, depending on the cost of fuel, may be significant.

In general, because the objective of our problem is to minimize fuel use, and battery discharge does not consume fuel, an optimal solution seeks to maximize battery dispatch; therefore, when demand is greater than the rate-capacity bounds, which occurs most often in the even-numbered scenarios, the likelihood and size of rate-capacity violations increases. If the opposite is true, which is the case for the odd-numbered scenarios, rate-capacity bounds may not be as restrictive, thereby reducing the likelihood and size of rate-capacity violations. Although there are situations in which choosing to use \( \text{E}^+ \) versus \( \text{PB}^+ \) is inconsequential, the scenario results indicate that failure to account for rate-capacity effects when considering four of the six batteries in Table 1 is nontrivial and it may invalidate the solution.

7. Conclusion

We present a detailed set of battery constraints to account for variable voltage and rate-capacity effects associated with steady-state dispatch for a hybrid power optimization problem. We provide a nonlinear physics-based set of constraints \( \text{N} \) in MINLP format, then derive a tractable, linear approximation \( \text{PB} \) that limits over-estimation error. A theoretical analysis examining discharge error relative to \( \text{N} \) as a function of SoC indicates that error in \( \text{E}^+ \) increases linearly with SoC and over-estimates performance by as much as 34% for a given battery type, while error in \( \text{PB}^+ \) is minimal and quadratic. To validate this analysis, we solve 12 scenarios and compare the resulting error in solutions to \( \text{E}^+ \) against that in \( \text{PB}^+ \) for a 6-month horizon at hourly fidelity. Results indicate that rate-capacity violations are most likely to occur when a battery's capacity is less than demand, leading to increased overestimation errors. Although rate-capacity errors in solutions to \( \text{E}^+ \) dominate constant voltage assumption errors in magnitude, assuming a constant voltage presents up to two percent error in magnitude per discharge. The maximum and average error in solutions to \( \text{PB}^+ \) compared to the error in solutions to \( \text{E}^+ \) is significantly less for 11 of 12 scenarios and similar in one. In particular, \( \text{E}^+ \) over-estimates discharge by as much as 16% in a number of scenarios compared to less than one percent across all scenarios in solutions to \( \text{PB}^+ \), but, more importantly, \( \text{E}^+ \) is exposed as a model that lacks detail, consistency, and has limited application.

Acknowledgements

The authors would like to thank Dr. Mark Spector, Office of Naval Research (ONR) for full support of this research effort under contract award #N000141310839. We also would like to acknowledge the support of the National Renewable Energy Lab (NREL) for its involvement in this project. We appreciate the meticulous comments of our two anonymous referees and of our associate editor.

Appendix A

We present the mathematical formulation of the objective function and system interoperability and technology constraints, which we call \( \text{P}^a \), necessary to minimize fuel-use for the steady state dispatch problem \( \text{P} \) presented in [33]. The combination of \( \text{P}^a \) with a set of battery constraints forms a hybrid power model...
(see Section 6). For each scenario, we fix procurement, which is similarly modeled in both (C) and (P) to the technologies and sizes depicted in Table 5, because our objective is to compare dispatch error between two sets of battery constraints in a hybrid power optimization model. Additionally, we do not model lifetime, because it is outside the scope of this paper, so all such associated parameters, variables, and constraints are omitted. We apply the same formulation characteristics to (P) as referenced in the paper.

Sets
\[ t \in T \] a single time period within the set of time periods

Parameters
\[ \tau \] length of one time period [h]
\[ d_t \] steady-state power demand in time period \( t \) [W]
\[ \eta \] power conversion efficiency of power exiting the battery
\[ \beta, \rho \] maximum power rating of the battery and the generator, respectively [W]
\[ b_t, c_t \] fuel consumption coefficients for generator power [gal/Wh, gal/h]
\[ \gamma_t \] power output of a PV panel in time period \( t \) [W/system]
\[ k \] fraction of PV power necessary to meet spinning reserve requirements
\[ x_{PV} \] integer number of PV systems [systems]

Variables
\[ P_t^G, P_t^2 \] aggregate power into and out of the battery in time period \( t \), respectively [W]
\[ P_t^F \] aggregate power out of the generator in time period \( t \) [W]
\[ P_t^{PV} \] aggregate power out of PV in time period \( t \) [W]
\[ \tilde{F}_t \] amount of fuel used in time period \( t \) [gal]
\[ G_t \] 1 if generator is operating in time period \( t \), 0 otherwise

Objective function
Minimize
\[ \sum_{t \in T} \tilde{F}_t \] (8a)

subject to

Constraints
\[ P_t^F - P_t^G = d_t \quad \forall t \in T \] (8b)
\[ P_t^{PV} \leq P_t^F \quad \forall t \in T \] (8c)
\[ \tilde{F}_t \leq \gamma_t P_t^{PV} \quad \forall t \in T \] (8d)
\[ P_t^{PV} = \gamma_t x_{PV} \quad t \in T \] (8e)
\[ P_t^F, P_t^G, P_t^{PV}, P_t^F \geq 0 \quad t \in T \] (8f)
\[ G_t \text{ binary } \quad t \in T \] (8g)

A.1. Detailed discussion of formulation

The objective function (8a) minimizes fuel use. Constraint (8b) ensures that the hourly dispatch strategy of generator, battery, and solar technologies meets demand. We apply a parameter \( \eta \) to account for conversion associated with battery discharge power from DC to AC. Due to the intermittence of solar power, constraint (8c) enforces “spinning reserve,” which ensures that a backup power source, either batteries and/or generators, is available to meet a fraction of the load supplied by PV. If a generator is running, constraint (8d) bounds output power to be less than a manufacturer-specified level. Constraint (8e) determines the amount of fuel used during time period \( t \). We limit the PV output power to the product of \( \gamma_t \) and size of the PV array in constraint (8f). Finally, constraints (8g) and (8h) enforce nonnegativity and binary restrictions, respectively.

References


